Adaptive Channel Estimation for Multiple-Input Multiple-Output Frequency Domain Equalization

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***Abstract*—In this paper we investigate adaptive channel estimation for frequency domain equalization (FDE) in a single carrier (SC) multiple-input multiple-output (MIMO) system. Two types of channel estimation methods are proposed, assuming uncorrelated and correlated frequency bins. The FDE coefficients are computed using the channel estimates. It is shown that our proposed structures significantly outperform the adaptive FDE without channel estimation at high SNR. In particular, the proposed LMS-SCE FDE approaches the performance of FDE with perfect channel state information (CSI), and has a fast convergence speed and reasonably low complexity.**

Keywords-Multiple-Input Multiple-Output (MIMO); Frequency Domain Equalization (FDE); Channel Estimation; Least-Mean- Square (LMS); Recursive-Least-Square (RLS).

I. INTRODUCTION

Frequency domain equalization (FDE) [1-5] has been shown to be an effective solution for frequency selective channels in a single-carrier (SC) system, and has been proposed in IEEE802.16 [1]. Adaptive FDE structures have been investigated in [6-7], where the equalizer coefficients are adaptively calculated based on the least-mean-square (LMS) or recursive-least-square (RLS) criterion, without channel estimation required. Another adaptive FDE structure is based on adaptive channel estimation [8] which is used to compute the equalizer coefficients. However, the work in [8] only assumed single-input single-output (SISO) and single-input multiple-output (SIMO) systems.

In this paper we investigate adaptive channel estimation for multiple-input multiple-output (MIMO) FDE, compared to adaptive MIMO FDE without channel estimation [7]. Our work is different in that we extend the work in [8] to a MIMO system, where the performance is more sensitive to the channel variation. Two types of adaptive channel estimation methods are proposed. The first one operates independently on each frequency bin and is referred to as *unstructured channel estimation* (UCE). The second one is called *structured channel estimation* (SCE) which utilizes the fading correlation between adjacent frequency bins. The channel estimates are updated by the LMS and RLS algorithms and are used for computing the FDE coefficients. Compared to the previous work on adaptive FDE without channel estimation [7], our proposed structures provide significant performance enhancement at high SNR. In particular, the proposed LMS-SCE FDE structure outperforms the other structures and approaches the performance of FDE with perfect channel state information (CSI). It also has a modest increase of complexity compared to LMS FDE without channel estimation, and a fast convergence speed.

II. SYSTEM MODEL

We investigate an uncoded MIMO system with K transmit antennas and L receive antennas. Let $d_k^{(p)}[i]$ denote the ith $(i=0,...M-1)$ data symbol in the pth block of M symbols transmitted by the kth $(k=1,...K)$ antenna, with unit average symbol energy and symbol period T. The channel memory is assumed to be N, and $h_{kl}^{(p)}[i]\,(i=0,...,N)$ denotes the channel impulse response (CIR) between the kth transmit antenna and the lth receive antenna over the pth block. Each block is appended with a length-N cyclic prefix which is discarded at the receiver to prevent the interblock interference (IBI) and to make the received block appear to be periodic with period M.

The mth $(m=0,...M-1)$ sampled signal of the pth received block at the lth $(l=1,...L)$ receive antenna is expressed as

$$x_l^{(p)}[m]=\sum_{k=1}^{K}\sum_{i=0}^{N}d_k^{(p)}[i]h_{kl}^{(p)}[m-i]+n_l^{(p)}[m] \tag{1}$$

Figure 1 illustrates the investigated MIMO FDE structure, where the received signals are transferred into the frequency domain by the FFT operation. The FFT of $x_l^{(p)}[m]$ is given by

$$X_l^{(p)}[m]=\sum_{k=1}^{K}D_k^{(p)}[m]H_{kl}^{(p)}[m]+N_l^{(p)}[m] \tag{2}$$

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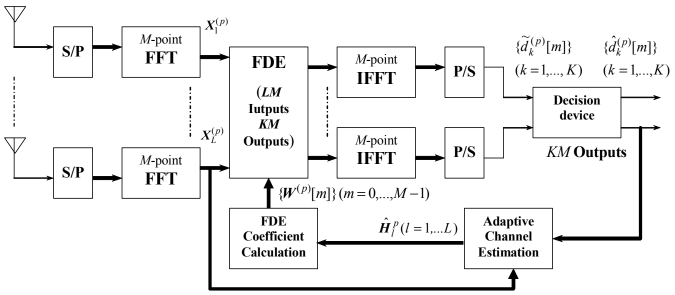

Figure 1. Block diagram of MIMO FDE with adaptive channel estimation

where $X_l^{(p)}[m] = \sum_{i=0}^{M-1} x_l^{(p)}[i] e^{-j2\pi im/M}$, $H_{kl}^{(p)}[m] = \sum_{i=0}^{N} h_{kl}^{(p)}[i] e^{-j2\pi im/M}$, $D_k^{(p)}[m] = \sum_{i=0}^{M-1} d_k^{(p)}[i] e^{-j2\pi im/M}$, $N_l^{(p)}[m] = \sum_{i=0}^{M-1} n_l^{(p)}[i] e^{-j2\pi im/M}$. Defining $\boldsymbol{D}^{(p)}[m] = \left[D_1^{(p)}[m] \ \ldots \ D_K^{(p)}[m]\right]$ and $\boldsymbol{H}_l^{(p)}[m] = \left[H_{1l}^{(p)}[m] \ \ldots \ H_{Kl}^{(p)}[m]\right]^T$, (2) becomes

$$X_l^{(p)}[m] = \boldsymbol{D}^{(p)}[m]\boldsymbol{H}_l^{(p)}[m] + N_l^{(p)}[m] \tag{3}$$

where $\boldsymbol{D}^{(p)}[m] = \left[D_1^{(p)}[m] \ \ldots \ D_K^{(p)}[m]\right]$, $\boldsymbol{H}_l^{(p)}[m] = \left[H_{1l}^{(p)}[m] \ \ldots \ H_{Kl}^{(p)}[m]\right]^T$.

The FDE coefficients are derived based on the minimum mean-square error (MMSE) criterion [5], i.e., to minimize

$$\Lambda = \sum_{k=1}^{K} E\left|\tilde{d}_k^{(p)}[i] - d_k^{(p)}[i]\right|^2 = \sum_{k=1}^{K} MSE_k^{(p)} \tag{4}$$

where $\tilde{d}_k^{(p)}[i]$ denotes the soft estimate of $d_k^{(p)}[i]$, and $MSE_k^{(p)} = E\left|\tilde{d}_k^{(p)}[i] - d_k^{(p)}[i]\right|^2$ is the MSE between $\tilde{d}_k^{(p)}[i]$ and $d_k^{(p)}[i]$.

III. ADAPTIVE CHANNEL ESTIMATION

We define a vector $\boldsymbol{X}_l^{(p)}$ as

$$\boldsymbol{X}_l^{(p)} = \tilde{\boldsymbol{D}}^{(p)}\boldsymbol{H}_l^{(p)} + \boldsymbol{N}_l^{(p)} \tag{5}$$

where $\boldsymbol{X}_l^{(p)} = \left[X_l^{(p)}[0] \ \ldots \ X_l^{(p)}[M-1]\right]^T$, $\boldsymbol{N}_l^{(p)} = \left[N_l^{(p)}[0] \ \ldots \ N_l^{(p)}[M-1]\right]^T$, and $\tilde{\boldsymbol{D}}^{(p)} = \begin{bmatrix} \boldsymbol{D}^{(p)}[0] & & \\ & \ddots & \\ & & \boldsymbol{D}^{(p)}[M-1] \end{bmatrix}$ which is of size $M \times KM$.

$\boldsymbol{H}_l^{(p)} = \left[\boldsymbol{H}_l^{(p)T}[0] \ \ldots \ \boldsymbol{H}_l^{(p)T}[M-1]\right]^T$ which can also be written as

$$\boldsymbol{H}_l^{(p)} = \tilde{\boldsymbol{F}}\boldsymbol{h}_l^{(p)} \tag{6}$$

where $\boldsymbol{h}_l^{(p)} = \left[h_{1l}^{(p)}[0] \ldots h_{1l}^{(p)}[N] \ldots h_{Kl}^{(p)}[0] \ldots h_{Kl}^{(p)}[N]\right]^T$ is the CIR vector of length $K(N+1)$ at the lth receive antenna. $\tilde{\boldsymbol{F}} = \left(\boldsymbol{F}_0^T \ldots \boldsymbol{F}_{M-1}^T\right)^T$ where $\boldsymbol{F}_m$ $(0 \le m \le M-1)$ is a $K \times K(N+1)$ block Toeplitz matrix defined as

$$\boldsymbol{F}_m = \begin{bmatrix} \boldsymbol{f}_m & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{f}_m & \ddots & \vdots \\ \vdots & \ddots & \ddots & \boldsymbol{0} \\ \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{f}_m \end{bmatrix} \tag{7}$$

with $\boldsymbol{f}_m = \left[e^{-j2\pi 0m/M} \ \ldots \ e^{-j2\pi Nm/M}\right]$.

We propose two types of adaptive channel estimation schemes. The first one is based on the assumption of independent frequency bins, referred to as *unstructured channel estimation* (UCE). The second one utilizes the correlation between adjacent frequency bins as shown in (6), and is referred to as *structured channel estimation* (SCE). The receiver first operates in the training mode where the training blocks are used to obtain the initial channel estimates. In the decision-directed mode, the transmitted symbols are replaced by the estimates of the data symbols.

A. LMS Unstructured Channel Estimation (LMS-UCE)

The LMS-UCE minimizes the cost function

$$J_{LMS-UCE}\left(\hat{\boldsymbol{H}}_l^{(p)}\right) = E\left\{\left\|\boldsymbol{X}_l^{(p)} - \tilde{\boldsymbol{D}}^{(p)}\hat{\boldsymbol{H}}_l^{(p)}\right\|^2\right\} \quad (l = 1,\ldots,L) \tag{8}$$

with respect to $\hat{\boldsymbol{H}}_l^{(p)}$ which is the estimate of $\boldsymbol{H}_l^{(p)}$. This produces

$$\hat{\boldsymbol{H}}_l^{(p+1)} = \hat{\boldsymbol{H}}_l^{(p)} + \mu\boldsymbol{E}_l^{(p)} \tag{9}$$

where μ is the step size and $\boldsymbol{E}_l^{(p)}$ is given by

$$\boldsymbol{E}_l^{(p)} = \tilde{\boldsymbol{D}}^{(p)H}\left[\boldsymbol{X}_l^{(p)} - \tilde{\boldsymbol{D}}^{(p)}\hat{\boldsymbol{H}}_l^{(p)}\right] \tag{10}$$

with $(\cdot)^H$ denoting Hermitian transposition.

B. RLS Unstructured Channel Estimation (RLS-UCE)

The RLS-UCE aims at minimizing the cost function

$$J_{RLS-UCE}\left(\hat{\boldsymbol{H}}_l^p\right)=\sum_{i=0}^{p}\lambda^{p-i}\left\|\boldsymbol{X}_l^{(i)}-\tilde{\boldsymbol{D}}^{(i)}\hat{\boldsymbol{H}}_l^{(p)}\right\|^2 \quad (l=1,\dots,L) \tag{11}$$

where λ $(0<\lambda<1)$ is the forgetting factor. $\hat{\boldsymbol{H}}_l^{(p)}$ satisfies the recursive equation

$$\hat{\boldsymbol{H}}_l^{(p+1)}=\hat{\boldsymbol{H}}_l^{(p)}+\boldsymbol{G}^{(p)}\boldsymbol{E}_l^{(p)} \tag{12}$$

where $\boldsymbol{E}_l^{(p)}$ is defined in (10) and

$$\boldsymbol{G}^{(p)}=diag\left(\boldsymbol{G}^{(p)}[0]\dots\boldsymbol{G}^{(p)}[M-1]\right) \tag{13}$$

is a block diagonal matrix, with $\boldsymbol{G}^{(p)}[m]$ expressed as

$$\boldsymbol{G}^{(p)}[m]=\frac{\boldsymbol{S}^{(p)}[m]}{\lambda+\boldsymbol{D}^{(p)}[m]\boldsymbol{S}^{(p)}[m]\boldsymbol{D}^{(p)^H}[m]} \tag{14}$$

where $\boldsymbol{S}^{(p)}[m]$ satisfies the recursion

$$\boldsymbol{S}^{(p+1)}[m]=\lambda^{-1}\left[\boldsymbol{I}-\boldsymbol{G}^{(p)}[m]\boldsymbol{D}^{(p)^H}[m]\boldsymbol{D}^{(p)}[m]\right]\boldsymbol{S}^{(p)}[m] \tag{15}$$

Note that $\boldsymbol{G}^{(p)}[m]$ and $\boldsymbol{S}^{(p)}[m]$ are independent of the index *l*, implying that they are the same at each receive antenna.

C. LMS Structured Channel Estimation (LMS-SCE)

The cost function of LMS-UCE is given by

$$J_{LMS-SCE}\left(\hat{\boldsymbol{h}}_l^{(p)}\right)=E\left\{\left\|\boldsymbol{X}_l^{(p)}-\tilde{\boldsymbol{D}}^{(p)}\tilde{\boldsymbol{F}}\hat{\boldsymbol{h}}_l^{(p)}\right\|^2\right\} \quad (l=1,\dots,L) \tag{16}$$

with respect to $\hat{\boldsymbol{h}}_l^{(p)}$ which is the estimate of $\boldsymbol{h}_l^{(p)}$. This produces

$$\hat{\boldsymbol{h}}_l^{(p+1)}=\hat{\boldsymbol{h}}_l^{(p)}+\mu\tilde{\boldsymbol{F}}^H\boldsymbol{E}_l^{(p)} \tag{17}$$

$$\hat{\boldsymbol{H}}_l^{(p+1)}=\hat{\boldsymbol{H}}_l^{(p)}+\mu\tilde{\boldsymbol{F}}\tilde{\boldsymbol{F}}^H\boldsymbol{E}_l^{(p)} \tag{18}$$

D. RLS Structured Channel Estimation (RLS-SCE)

The objective of RLS-SCE is to minimize the cost function

$$J_{RLS-SCE}\left(\hat{\boldsymbol{h}}_l^{(p)}\right)=\sum_{i=0}^{p}\lambda^{p-i}\left\|\boldsymbol{X}_l^{(i)}-\tilde{\boldsymbol{D}}^{(i)}\tilde{\boldsymbol{F}}\hat{\boldsymbol{h}}_l^{(p)}\right\|^2 \quad (l=1,\dots,L) \tag{19}$$

This however requires prohibitive complexity as no recursion can be used to compute the inverse of a matrix. Therefore, we do not consider this method in the following.

IV. Complexity Analysis

The computational complexity is approximately evaluated by counting the number of complex multiplications per detected block of signals. The complexity of FDE with LMS-UCE, RLS-UCE and LMS-SCE is shown in Table I, compared to that of LMS FDE and RLS FDE [7], where the FDE coefficients are updated adaptively without channel estimation. With *K*=4 transmit antennas, *L*=4 receive antennas, and a data block size *M*=64, their overall normalized complexity is demonstrated in Table II. It can be derived that LMS FDE requires the least complexity and RLS-UCE FDE requires the most complexity. The other three FDE schemes have similar complexity.

TABLE I. Computational Complexity per detected block

<table>
<tr><th></th><th>Channel Estimation</th><th>FDE Coefficients</th><th>FFT+FDE
+IFFT</th></tr>
<tr><td>LMS-UCE FDE</td><td>$(0.5\log_2 M+2L)KM$</td><td rowspan="3">$L^3M/3+2KL^2M$</td><td rowspan="5">$0.5(K+L)M(\log_2 M)$
$+KLM$</td></tr>
<tr><td>RLS-UCE FDE</td><td>$(0.5\log_2 M+2L)KM$
$+(K+L+3)K^2M+KM$</td></tr>
<tr><td>LMS-SCE FDE</td><td>$(0.5\log_2 M+2L)KM$
$+KLM\log_2 M$</td></tr>
<tr><td>LMS FDE</td><td>0</td><td>$(0.5\log_2 M+2L)KM$</td></tr>
<tr><td>RLS FDE</td><td>0</td><td>$(0.5\log_2 M+2L)KM$
$+(L+K+3)L^2M+LM$</td></tr>
</table>

TABLE II. NORMALIZED Computational Complexity PER DETECTED BLOCK WITH *K*=4, *L*=4 AND *M*=64

LMS-UCE FDE	RLS-UCE FDE	LMS-SCE FDE	LMS FDE	RLS FDE
336%	621%	339%	100%	385%

V. Simulation Results

We use simulation results to show the performance of FDE the three adaptive channel estimation methods shown in Tables I-II, with K=4 transmit and L=4 receive antennas. Each data block consists of M=64 QPSK symbols, with a data rate of 1.25 Mbps. The transmit and receive filters use a raised-cosine pulse with a roll-off factor of 0.35. We consider a typical urban environment where the RMS delay spread is $1\mu s$. The overall channel memory is $N = 6$, lumping the effects of the transmit filter, receiver filter and real channel. The SNR is defined as the spatial average ratio of the received signal power to noise ratio.

In the following, the step sizes we use for LMS-UCE FDE, LMS-SCE FDE and LMS FDE are $\mu = 1\times10^{-4}$, $\mu = 8\times10^{-4}$ and $\mu = 1\times10^{-4}$, respectively. The forgetting factor for both RLS-UCE FDE and RLS FDE is set to $\lambda = 0.8$.

Figure 2 shows the BER performance of the five FDE structures in Tables I-II with 10 training blocks, compared to the case with perfect CSI. All the FDE structures with adaptive channel estimation significantly outperform the adaptive FDE structures without channel estimation at high SNR. In particular, LMS-SCE FDE provides the best performance, approaching the performance of FDE with perfect CSI. It has a performance gain of 3.8dB over LMS FDE at BER=10^{-3}, with a modest increase of complexity.

Figure 3 illustrates the learning curves for the five FDE structures, in terms of MSE versus the number of training blocks. The MSE is defined in (4) with an SNR of 15 dB. It can be seen that RLS-UCE FDE has the fastest convergence speed with only 3 training blocks required, at the cost of complexity. LMS-SCE FDE has the lowest MSE in the steady state close to that of FDE with perfect CSI, with only 5 training blocks required. LMS FDE and RLS FDE without channel estimation provide much worse performance than the channel estimation based FDE structures.

VI. CONCLUSION

Two types of adaptive channel estimation methods have been proposed for MIMO FDE, which significantly outperform adaptive FDE without channel estimation at high SNR. In particular, the LMS-SCE FDE has the best performance close to the performance of FDE with perfect CSI. It also has a fast convergence speed, and a modest increase of complexity compared to LMS FDE without channel estimation.

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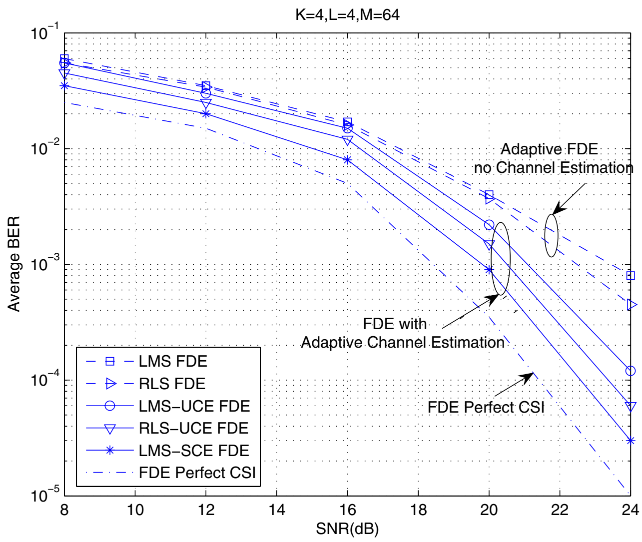

Figure 2. MIMO FDE with K=4, L=4 and 10 training blocks

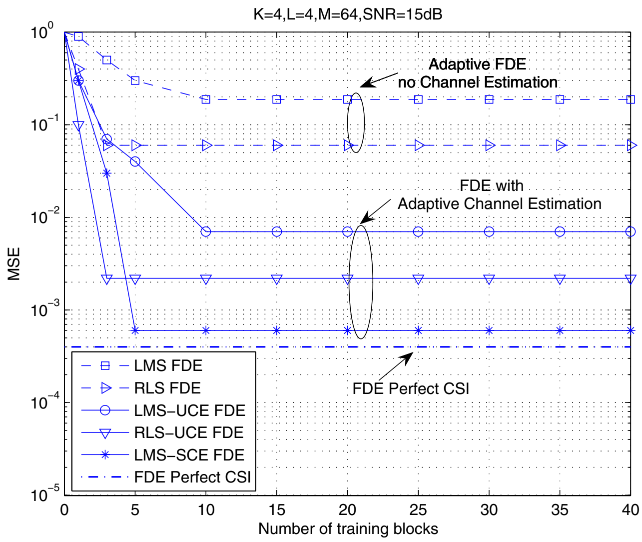

Figure 3. Learning curves for MIMO FDE with K=4, L=4 and SNR=15dB